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THE ELEMENTS OF THE CONIC SECTIONS, AS PREPARATORY TO THE READING OF SIR I. NEWTON'S PRINCIPIA.

BY THE

REV. S. VINCE, A.M.

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ERRATA.

Page 30. line 3. *for* coincides, *read* becomes equal; l. 4. *for* with, *read* to, *and for* and conſequently, *read* which coincides.

Page 51. l. 14. *for* coincides, *read* becomes equal; l. 15. *for* with, *read* to, *and for* and conſequently, *read* which coincides.

Of the PARABOLA.

DEF. I. IF a point S* be aſſumed in the line LZ, and ELD be drawn perpendicular to it; then if a line MW moves parallel to LZ interſecting another line SR revolving about S, ſo that MP may be always equal to SP, the curve paſſing through all the interſections P is called a *parabola*.

* Fig. 1.

Cor. Hence the curve cuts LZ in a point A which biſects LS, and A is called the *vertex*.

2. The point S is called the *focus*.

3. Any line PW parallel to AZ is called a *diameter*, and AZ is called the *axis*.

4. The line ED is called the *directrix*.

5. If PN be drawn perpendicular to AZ, AN is called an *abſciſſa*, and PN an *ordinate* to the axis.

6. If QV be drawn parallel to a tangent at P, PV is called an *abſciſſa*, and QV an *ordinate* to the *diameter* PW.

7. If BSC be drawn through S perpendicular to AZ, it is called the *parameter* or *latus rectum* to the axis.

PROP. I.

The diſtance $SP = AN + AS$.

For SP being $= PM$ and $MPNL$ a parallelogram by conſtruction, $\therefore SP = MP = NL = NA + AL =$ (as $AL = AS$ by cor. to def. 1.) $AN + AS$.

PROP. II.

The diſtance $AS = \frac{1}{4}$ *of the latus rectum.*

For when P comes to B, N coincides with S, and then by pr. 1. $SB = SA + SA = 2SA$, $\therefore SA = \frac{1}{2}SB = \frac{1}{4}BC$.

PROP.

PROP. III.

The ſquare of any ordinate PN = BC × AN.

For by pr. 1. $SP = NA + AS =$ (by pr. 2.) $NA + \frac{1}{4}BC$; and $NS = NA - SA = NA - \frac{1}{4}BC$, hence (by Euc. 47.1.) $NP^2 = PS^2 - SN^2 = \overline{NA + \frac{1}{4}BC}^2 - \overline{NA - \frac{1}{4}BC}^2 = BC \times AN$.

Cor. 1. Hence as $4SA = BC$, $NP^2 = 4SA \times AN$.

Cor. 2. Hence alſo, BC being conſtant, AN varies as NP^2.

PROP. IV.

A tangent PT[*] *to the point* P *biſects the angle* MPS.

Aſſume Pp an indefinitely ſmall part of the curve, which may therefore be conſidered as coincident with the tangent; join Sp, and draw pm parallel to PM, and from P draw Pa, Pb, perpendicular to mp, Sp reſpectively; then as $MP = SP$ and $mp = sp$, their differences ap, bp are equal, hence in the right

[*] Fig. 2.

angled $\triangle$s, Pap, Pbp, we have $ap = bp$, and Pp common, $\therefore \angle Ppa = \angle Ppb$, and conſequently, as mp is parallel to MP, and Sp infinitely near to SP, the $\angle MPT = \angle SPT$.

Cor. 1. A tangent at the point A is perpendicular to the axis.

Cor. 2. The lines MP, TS being parallel, the $\angle MPT = \angle PTS$, $\therefore \angle PTS = SPT$, and hence $SP = ST$.

Cor. 3. As VQ is parallel to PT, $\angle TPX = \angle PXV$* and $\angle PVX = \angle MPT$, but as $\angle MPT = \angle TPX$, $\therefore PVX = PXV$, and conſequently $PX = PV$.

* Fig. 1.

PROP. V.

The ſubtangent TN *is* = 2 NA.

For by cor. 2. pr. 4. $TS = SP =$ (by pr. 1.) $AN + AS$, take away AS from both, and $TA = AN$, $\therefore TN = 2NA$.

Cor. If PG be perpendicular to PT, then by ſim. $\triangle$s TNP, NPG, $TN : PN$

$:: PN : NG = \frac{PN^2}{TN} =$ (as by cor. 1. pr. 3. $PN^2 = AN \times 4SA)\ \frac{AN \times 4SA}{2AN} = 2SA$ $= \frac{1}{2}BC = \frac{1}{2}$ the latus rectum.

LEM. I.

If VQ* *be produced to meet* TZ *in* O, *and* VG *be drawn perpendicular to* AZ, *then will* OG = 2AN.

[*] Fig. 3.

For the Δs TPN, OVG being ſimilar, and $NP = GV$, $\therefore OG = TN = 2AN$.

PROP. VI.

The ſquare of VQ = 4PS × PV.

Through Q draw EQD perpendicular to AZ meeting WP (produced if neceſſary) in D. Then $OE = OG - GE =$ (by lem. 1.) $2AN - GE$, alſo $AE = AN + NG - GE$; and by ſimilar Δs TNP, OEQ, TN^2 $(4AN^2)$: NP^2 $(4AS \times AN)$ :: OE^2 $(\overline{2AN - GE}^2)$: $EQ^2 = \frac{\overline{2AN - EG}^2 \times AS}{AN}$, but by cor. 1. pr. 3. $EQ^2 = 4AS \times AE = 4AS \times \overline{AN + NG - GE}$; which being made equal to the other value of EQ^2, and the common quantities taken from both ſides of the equation, we have $4AN \times NG = GE^2$; alſo by ſimilar Δs OGV, VDQ, OG^2 $(= 4AN^2$ by lem. 1.) : GV^2 $(= NP^2 = 4AS \times AN)$ :: VD^2 $(= GE^2 = 4AN \times NG)$: $DQ^2 = 4AS \times NG$, hence $QV^2 = VD^2 + DQ^2 = 4AN \times NG + 4AS \times NG = \overline{4AN + 4AS} \times NG = 4SP \times PV$.

If Q falls on the other ſide of P, then $OE = OG + GE$, and $AE = AN + NG + GE$, in which caſe the ſame concluſion reſults as above.

Cor. If QI be drawn parallel to PV, $4SP \times QI = PI^2$.

Def. 8. $4SP$ is called the *latus rectum* or *parameter* to the diameter PW.

PROP. VII.

If a perpendicular SY* *be let on the tangent, then will* SP : SY :: SY : SA. * Fig. 4.

For, as by cor. 2. pr. 4. $SP = ST$, the perpendicular SY muſt biſect TP, or $TY = YP$, but by pr. 5. $TA = AN$, $\therefore AY$ is parallel to PN, and conſequently perpendicular to the axis. Since therefore SY is perpendicular to TY, and YA perpendicular to TA, the $\triangle\ SYA$ is ſimilar to SYT or SYP, $\therefore SP : SY :: SY : SA$.

Cor. 1. Hence $SP : SA :: SP^2 : SY^2$.

Cor. 2. Hence, as $SP = \frac{SY^2}{SA}$, and SA is conſtant, SP varies as SY^2.

Cor. 3. As $PS \times SA = SY^2$, $4SY^2 = 4SA \times PS = BC \times PS$.

LEM. II.

* Fig. 5. *If* PR* *be a tangent to the circle* PVQ *at* P, *and* RQ *be drawn parallel to the chord* PV, *then will* $PV = \frac{\overline{\text{arc } PQ}^2}{QR}$, *when the arc* QP *is diminiſhed ad infinitum.*

For the △s PRQ, QPV being ſimilar, (the $\angle PQR$ being $= \angle QPV$ on account of the parallel lines QR, PV, and the $\angle RPQ = \angle PVQ$ in the alternate ſegment,) $\therefore QR : QP :: QP : PV = \frac{QP^2}{QR} =$ in the evaneſcent ſtate of the arc PQ (as the chord and arc become ultimately equal by lem. 7. Prin. Newt.) $\frac{\overline{\text{arc } PQ}^2}{QR}$.

PROP. VIII.

* Fig. 6. *The chord of curvature* Pv* *paſſing through the focus* = 4PS.

Let Pv be the chord of curvature paſſing through S, and draw the ordinate QXV and alſo QR parallel to Pv. Now by pr. 6. $4PS = \frac{QV^2}{PV}$, which is equal, if Q moves up to P, in the ultimate ſtate of the arc QP, $\frac{QP^2}{PV}$. But the arc QP coincides in its ultimate ſtate with the circle of curvature, and by lem. 2. the chord Pv of that circle $= \frac{QP^2}{QR}$ $=$ (as $QR = PX = PV$ by cor. 3. pr. 4.) $\frac{QP^2}{PV}$, therefore $Pv = 4PS$.

PROP. IX.

If there be two parabolas AY*, AZ, *deſcribed on the ſame axis, to which* NPQ *is drawn perpendicular, then if the lines* PS, QS, *be drawn to any point* S *in the axis, the area* AQS *will be to* APS *in a given ratio.*

* Fig. 7.

For by cor. 2. pr. 3. QN^2 varies as AN, alſo PN^2 varies as AN, hence QN^2 varies as PN^2, and conſequently QN varies as PN. Draw qpn indefinitely near to QPN, and then will the areas $NQqn$, $NPpn$ be the increments of the areas AQN, APN, which will be to each other as QN to PN; hence the areas AQN, APN beginning together, and always increaſing in the given ratio of QN to PN, the areas themſelves will be in that ratio. But the $\triangle SQN$ is to $\triangle SPN$ as QN to PN, and hence $AQN - SQN$ is to $APN - SPN$, that is, AQS is to APS as QN to PN, or in a given ratio. If N be between A and S, then $AQS = AQN + SQN$ and $APS = APN + SPN$.

PROP. X.

* Fig. 28. *Let* WP* *be a diameter of the parabola* APY, *then if a tangent be drawn to any point* Q *meeting the diameter produced in* t, *a tangent* PK *to the point* P *will biſect* tQ.

For draw the ordinate QV, and vq indefinitely near to it, alſo Qd parallel to Pv; then by pr. 6. $4PS = \frac{QV^2}{PV}$, and hence by the ſame pr. $\frac{VQ^2}{PV} \times Pv = vq^2$, but $Pv = PV + Vv = PV + Qd$, and $vq = vd + dq = VQ + dq$, therefore $\frac{VQ^2}{PV} \times \overline{PV + Qd} = \overline{VQ + dq}^2$, hence, by multiplying both ſides of the equation by PV, and taking from them the common quantities, neglecting $PV \times dq^2$, as indefinitely leſs than any of the other quantities, and dividing both ſides by $VQ \times dq$, we ſhall have $2PV = \frac{VQ \times Qd}{dq}$, but the Δs qdQ, QVt being ſimilar we have $dq : dQ :: QV : Vt = \frac{QV \times Qd}{dq}$, therefore $2PV = Vt$ or $PV = Vt$, and hence, as PK is parallel to QV, $tK = KQ$.

PROP. XI.

If a cone HXY* *be cut by a plane* FZW *parallel to a plane touching the ſide of the cone* XH, *the ſection* FZW *will be a parabola.*

* Fig. 8.

Let $AECV$ be a ſection of the cone parallel to the baſe, whoſe diameter (the ſection being a circle) interſects the axis FR of the figure FZW in B, and whoſe circumference cuts the curve FZ terminating the ſection in E, and join BE; then by the property of the circle $AB \times BC = BE^2$. Draw GF parallel to ABC and then $AB = GF$ a conſtant quantity;

and by ſimilar $\triangle$s HGF, FBC, $HG : GF :: FB : BC = \frac{GF \times FB}{HG}$, therefore $\frac{GF^2 \times FB}{HG} = GF \times BC = AB \times BC = BE^2$, or $\frac{GF^2}{HG} \times FB = BE^2$, but GF and HG being conſtant quantities, the abſciſſa FB varies as the ſquare of ordinate BE, and conſequently the curve is a parabola as appears by cor. 2. pr. 3.

Hence the parabola is called a conic ſection.

Of

Of the ELLIPSE.

DEF. 1. IF two right lines Sa*, Hb, revolve about two fixed points S, H, and continually interſect each other in a point P, ſo that $SP + HP$ may be a conſtant quantity, the curve deſcribed by the point P is called an *ellipſe*.

* Fig. 9.

2. The points S and H are called *foci*.

3. The right line AM paſſing through the foci and terminated by the curve at A and M, is called the *axis major*.

4. If AM be biſected in C (which is called the *center*) and BCN be drawn perpendicular to the axis major and terminated by the curve, it is called the *axis minor*.

5. A right line LST paſſing through the focus S perpendicular to the major axis and terminated by the curve, is called the *latus rectum.*

* Fig. 10. 6. If PCG* be a right line paſſing through the center, it is called a *diameter*. And if DCF be drawn parallel to a tangent at P, it is called a *conjugate diameter*.

7. If QV be drawn parallel to a tangent at P, it is called an *ordinate*, and GV, PV are called *abſciſſas.*

PROP. I.

* Fig. 9. *The ſum of the two lines* AP*, HP, *is equal to the major axis* AM, *or to* $2AC$.

For conceiving the deſcribing the point P to come to A and M, we have in the former caſe the ſum of the lines drawn from S and H equal to $HA + AS = SH + 2AS$, and in the latter equal to $SH + 2HM$; hence by def. 1. $SH + 2AS = SH + 2HM$, conſequently $SA = HM$, and hence $HP + PS$ $(= HA + AS) = HA + HM$ or $2AC$.

Cor. 1. Hence $SC = CH$.

Cor. 2. Hence alſo by tranſpoſition $HP = 2AC - SP$.

PROP. II.

The right line SB = AC.

For the $\triangle$s HCB, SCB, have $SC = CH$ by cor. 1. p. 1. BC common, and $\angle SCB = \angle HCB$, $\therefore SB = BH$, but by pr. 1. $SB + BH = 2AC$, $\therefore 2SB = 2AC$, and conſequently $SB = AC$.

Cor. Hence $AS \times SM = BC^2$. For $BC^2 = BS^2 - SC^2 = AC^2 - SC^2 = \overline{AC + SC} \times \overline{AC - SC} = \overline{MC + SC} \times \overline{AC - SC} = MS \times SA$.

PROP. III.

The latus rectum LT *is a third proportional to the axis major* AM *and the axis minor* BN.

By cor. 2. pr. 1. $HL = 2AC - SL$, hence $HS^2 + SL^2$ $(= LH^2)$ $= 4AC^2 - 4AC \times SL + SL^2$, $\therefore HS^2 + 4AC \times SL = 4AC^2 =$ by pr. 2. $4SB^2$, hence $4AC$

$\times LS$

$\times SL =$ (becauſe $2SC = SH$) $4SB^2 - 4SC^2$ (SH^2) $= 4BC^2$, $\therefore 2AC : 2BC :: 2BC : 2SL$, or $AM : BN :: BN : LT$.

PROP. IV.

* Fig. 11. *Two right lines* SP, HP*, *drawn from the foci to the curve, make equal angles with the curve.*

For draw SQ indefinitely near to SP and join HQ, and draw Pa, Qb perpendicular to SQ and HP reſpectively, then will Qa be the increment of SP, and Pb the decrement of PH, which, as $SP + HP$ is conſtant, muſt be equal to each other; alſo the $\angle$s PaQ, PbQ are each right angles, and the ſide PQ is common to the two $\triangle$s PaQ, PbQ, $\therefore \angle bPQ = \angle aQA =$ (as the $\angle PSQ$ is indefinitely ſmall) $\angle SPX$.

Cor. 1. Hence if HP be produced to Z, a tangent at P will biſect the $\angle ZPS$. For let VPW be a tangent at P, then as the $\angle SPV$ or SPX is $= \angle HPQ$, and $\angle HPQ$ is vertical to $\angle ZPV$, the $\angle SPV = \angle ZPV$.

* Fig. 12. Cor. 2. Hence the part PE* of the

line PS intercepted between any point P and the conjugate diameter DCK is equal to AC. For draw HI parallel to DCK and PF perpendicular to it, and conſequently perpendicular to HI interſecting it in X. Then as $\angle SPV = \angle HPW$, the $\angle IPX = \angle HPX$, and PX is common to the two right $\angle$'d Δs PXI, PXH, hence $PI = PH$. But as $SC = CH$ and CE is parallel to HI, $SE = IE$, or $IE = \frac{SI}{2}$, but as $PI = PH$, $PI = \frac{PI + PH}{2}$, hence $IE + PI$ or $PE = \frac{SP + PH}{2} =$ by pr. 1. to AC.

PROP. V.

If on the major axis AM* *of an ellipſe there be deſcribed a circle* AQMR, *and from each of the foci perpendiculars be drawn to a tangent to the ellipſe at the point* P, *they will meet it in the periphery of the circle.*

* Fig. 13.

For let SY, HZ be drawn from the two foci perpendicular to the tangent, join YC and produce HP to W, making $PW = PS$ and join WY. Then by cor. 1. to pr. 4. $\angle WPY = \angle SPY$, and as $PS = PW$ and PY is common to the $\triangle$s SPY, WPY, $WY = YS$ and $\angle PYS = \angle PYW$, but $\angle PYS$ is a right angle, $\therefore PYW$ is alſo a right angle, and conſequently SYW is a ſtraight line. Now SY being $= YW$, and by cor. 1. pr. 1. $SC = CH$, CY is parallel to HW, hence $SC : CY :: SH : HW$, but $SC = \frac{1}{2}SH$, $\therefore CY = \frac{1}{2}HW = \frac{HP + PS}{2} =$ (by pr. 1.) $\frac{1}{2}AM = AC$; hence CY being $= CA$, Y muſt always be found in the periphery of a circle whoſe center is C and radius CA. In the ſame manner it may be proved for the other perpendicular HZ, that Z is in the periphery.

PROP. VI.

The rectangle under the perpendiculars SY *and* HZ, *is equal to* BC^2.

For produce ZH to meet the circle in E and join CE, then by the laſt pr. Z being in the periphery of the circle and the $\angle HZY$ a right $\angle$, the $\angle YZE$ muſt be in a ſemicircle, and conſequently YCE muſt be a ſtraight line. Hence in the Δs YCS, ECH, $YC = EC$, $SC = HC$ and $\angle YCS = \angle ECH$, $\therefore EH = YS$; but by the property of the circle $ZH \times HE = MH \times HA =$ (by cor. to pr. 2.) BC^2, $\therefore ZH \times SY = BC^2$.

Cor. 1. Becauſe by pr. 4. $\angle SPY = \angle HPZ$, the Δs SPY, HPZ are ſimilar, $\therefore SP : SY :: HP : HZ = \frac{SY \times HP}{SP}$, hence $\frac{SY^2 \times HP}{SP} = SY \times HZ = BC^2$, conſequently $SY^2 = BC^2 \times \frac{SP}{HP}$.

Cor. 2. Hence, as BC is conſtant, SY varies as $\sqrt{\frac{SP}{HP}}$, but $HP =$ (by cor. 2. pr. 1.) $2AC - SP$, $\therefore SY$ varies as $\sqrt{\frac{SP}{2AC - SP}}$, or $\frac{1}{SY}$ varies as $\sqrt{\frac{2AC - SP}{SP}}$.

Cor. 3. Becauſe $4SY^2 = \frac{4BC^2 \times SP}{2AC - SP}$, and by pr. 3. $4BC^2 = 2AC \times L$, therefore $4SY^2 = \frac{2AC \times L \times SP}{2AC - SP}$, hence $L \times SP : 4SY^2 :: L \times SP : \frac{2AC \times L \times SP}{2AC - SP} :: 2AC - SP : 2AC$, conſequently, as $2AC - SP$ is leſs than $2AC$, $L \times SP$ is leſs than $4SY^2$.

PROP. VII.

*Fig. 14. *If* PN* *be perpendicular to* AM, *then will* $AN \times NM : NP^2 :: AC^2 : BC^2$.

Take $AD = PS$, then $PS = AC - CD$, and conſequently by pr. 1. $PH = AC + CD$; alſo $NS = CS - CN$ and $HN = CH + CN = CS + CN$; hence by Eucl. 47. 1. $\overline{AC + CD}^2 = PN^2 + \overline{CS + CN}^2$, and $\overline{AC - CD}^2 = PN^2 + \overline{CS - CN}^2$, ſubtract this latter equation former, and we have $AC \times CD = CS \times CN$, $\therefore CD = \frac{CS \times CN}{AC}$; hence $PS =$

AC

$$AC - \frac{SC \times CN}{AC} = \frac{AC^2 - SC \times CN}{AC},$$

∴ by Eucl. 47.1. (as $NS = CS - CN$,)

$$\frac{AC^4 - 2AC^2 \times SC \times CN + SC^2 \times CN^2}{AC^2}$$

$= PN^2 + CS^2 - 2CS \times CN + CN^2$, multiply both ſides of the equation by AC^2, and we get by tranſpoſition, $AC^2 \times PN^2 = AC^4 - AC^2 \times CN^2 - CS^2 \times AC^2 + SC^2 \times CN^2 = \overline{AC^2 - CN^2} \times \overline{AC^2 - CS^2} =$ (as $AC = SB$) $\overline{AC^2 - CN^2} \times \overline{SB^2 - CS^2}\ (BC^2) = \overline{AC + CN} \times \overline{AC - CN} \times BC^2 = MN \times AN \times BC^2$, and conſequently $MN \times AN : PN^2 :: AC^2 : BC^2$.

Cor. 1. Hence if a circle AQM* be deſcribed on AM as a diameter, and NP be produced to Q; NP ſhall be in a given ratio to NQ; for as $AC^2 : BC^2 :: MN \times AN : PN^2$, and AC and BC are conſtant, $MN \times AN$ varies as PN^2; but by the property of the circle $AN \times MN$ is equal to QN^2, ∴ QN^2 varies as PN^2, and conſequently QN varies as PN.

* Fig. 15.

Cor. 2. Hence $QN : PN ::$ axis major :

jor : axis minor; for when PN coincides with BC, QN becomes equal to $RC = AC$, and conſequently $PN : QN :: BC : RC$ or $AC :: 2BC$ or $BG : 2AC$ or AM.

Cor. 3. Draw qpn indefinitely near to QPN, and then $NPpn$, $NQqn$ will be the increments of the areas APN, AQN reſpectively, which, as the baſes Nn are the ſame, will be to each other as their altitudes NP, NQ, or by the laſt cor. as $BG : AM$, hence the areas APN, AQN beginning together and increaſing always in the given ratio of $BG : AM$, the areas generated muſt, by compoſition, be in the ſame ratio, $\therefore APN : AQN :: BG : AM$.

Cor. 4. Join QH, PH, then the △s HNP, HNQ are to each other as $NP : NQ$, or as $BG : AM$, from which and the laſt cor. we have the area $APH : AQH :: BG : AM$. And the ſame muſt evidently be true if H be aſſumed any where in the axis major.

Cor. 5. Hence, as the whole area (A) of the ellipſe is to the area (B) of the circle, as the axis minor (a) is to the axis major (b), $\therefore A = \frac{B \times a}{b}$, but B varies as b^2, $\therefore A$ varies as $b \times a$; that is, the area of an ellipſe varies as the rectangle under the axis major and minor; but the area of a ▭ circumſcribing the ellipſe about the axis major and minor, varies in the ſame ratio, hence the area of every ellipſe is in a given ratio of the area of the ▭ about the major and minor axis.

Cor. 6. If $L =$ the latus rectum, then by pr. 3. $AM : BG :: BG : L$, hence $AM : L :: AM^2 : BG^2 :: AC^2 : BC^2 ::$ (by this pr.) $MN \times AN : PN^2$, conſequently L

$$= \frac{AM \times PN^2}{MN \times AN} = \frac{2AC \times PN^2}{MN \times AN}.$$

Cor. 7. If the major axis of an ellipſe becomes infinite, the ellipſe at all finite diſtances from the extremities of the major axis becomes a parabola. For if AM becomes infinite and AN be aſſumed finite, NM will be conſtant, and conſequently by cor. 1. AN will vary as NP^2, and therefore by cor. 2. pr. 3. of the parabola, the curve is a parabola.

PROP. VIII.

If a circle be deſcribed on the major axis AM* *of an ellipſe, and* NP *be produced to meet it at* Q, *the tangents to the circle and ellipſe at the points* Q *and* P *will interſect the axis* MA *produced in the ſame point* T.

* Fig. 16.

For draw QT a tangent to the circle at Q and join TP, and if it be not a tangent to the ellipſe, let it cut it in ſome other point K, and draw the ordinate LK, which produce to meet the circle at I, and its tangent at O, then the $\triangle$s TNP, TLK, and TNQ, TLO are ſimilar; hence

$$TN : TL :: NP : LK,$$
and $$TN : TL :: NQ : LO,$$
$$\therefore\ NP : LK :: NQ : LO,$$
$\therefore$ alter. $$NP : NQ :: LK : LO,$$
but $NP : NQ :: LK : LI$ by cor. 1. pr. 7. $\therefore LO = LI$, which is abſurd; hence the line TP does not cut the ellipſe, and conſequently is a tangent to it.

Cor. By Eucl. B. vi. p. 8. the $\triangle$ CNQ, CQT are ſimilar, $\therefore CN : CQ (CA) :: CQ (CA)$

$(CA):CT$; or, as $CA=CM$, $CN:CM::CM:CT$.

L E M. III.

If a right line AB* *be projected upon a plane* XY, *by lines drawn from every point of it perpendicular to the plane, the length of the line to be projected will be to the length of the line into which it is projected :: radius : coſine of inclination.*

* Fig. 17.

For draw Aa, Bb perpendicular to XY, and join ab, then will ab be the projected line; draw AC parallel to XY, and we have $AB:AC$ or ab :: radius : coſine $\angle BAC$.

Cor. 1. Hence if the inclination be given, AB is to ab in a given ratio.

Cor. 2. Hence the area of any plane figure will be to the area of its projection as radius : coſine of inclination.

L E M. IV.

If a circle be projected in the ſame manner, the figure into which it is projected will be an ellipſe.

For

* Fig. 18. For let AZM* be a ſemicircle, and conceive it to be projected into AzM. Draw NP, Np perpendicular to the common interſection AM of the two figures, then is the angle PNp the inclination, and conſequently $PN : pN ::$ radius : coſine of inclination, hence PN varies as pN, but $AN \times NM = NP^2$, conſequently $AN \times NM$ varies as pN^2, and therefore by cor. 1. pr. 7. the curve $ApzM$ is an ellipſe.

PROP. IX.

If an ordinate be drawn to any diameter of an ellipſe, then will the rectangle under the abſciſſas be to the ſquare of the ordinate in a given ratio.

* Fig. 19. Conceive the circle $APMG$* be projected into the ellipſe $ApMg$; let PG be any diameter, and QV an ordinate perpendicular to it, and let pg and qv be their projection. Then as PV, VG are projected into pv, vg, theſe quantities will (by cor. 1. lem. 3.) be to the former in a given ratio, hence $pv \times vg$ varies as $PV \times VG$, but $PV \times VG = QV^2$, $\therefore pv \times vg$ varies as QV^2; but as QV is projected

ed into qv; QV varies as qv, or QV^2 varies as qv^2, conſequently $pv \times vg$ varies as qv^2; and as QV is parallel to a tangent at V, and parallel lines are projected into parallel lines, therefore qv is parallel to a tangent to the ellipſe at p, for a tangent to the circle muſt evidently be projected into a tangent to the ellipſe.

Cor. 1. Hence, as $PC = CG$, therefore $pC = Cg$, conſequently every diameter is biſected in the center.

Cor. 2. Hence if DCK be drawn parallel to qv, $pv \times vg : qv^2 :: pC^2 : CD^2$.

Cor. 3. Hence the relation between the abſciſſas and ordinate being the ſame when referred to any diameter, as when referred to the major axis, the concluſion deduced in pr. 8. and its cor. relative to the interſection of the tangent and major axis, muſt be true for the interſection of the tangent and any diameter, it being evident that the demonſtration does not at all depend on the ordinates NP*, LK being perpendicular to AM, but only on their being in the ſame ratio to each other, as NQ to LI, and therefore the ſame concluſions muſt follow if

* Fig. 16.

NP, LK be oblique to AM, which is the caſe when AM be any other diameter than the major axis.

PROP. X.

All the parallelograms circumſcribing an ellipſe are equal.

For all the ſquares deſcribed about the ſame circle are equal, and if the circle be projected into an ellipſe, every ſquare will be projected into a circumſcribing parallelogram; for the oppoſite ſides of the ſquare being parallel and tangents to the circle, they will be projected into parallel lines, which will be tangents to the ellipſe. Now as every ſquare is by cor. 2. lem. 3. diminiſhed by projection in the ratio of radius : coſine of inclination, or in a given ratio, and the areas of all the ſquares are equal, the areas of all the parallelograms muſt be equal.

Cor. 1. From hence, and cor. 5. pr. 7. the area of the ellipſe varies as the area of the circumſcribing ▭ $abcd$*, and as the area $aPCD$ of a fourth part of the ▭ is equal to $DC \times PF$, (PF being perpendicular to DCK) the whole ▭ varies as

* Fig. 21.

as $DC \times PF$; therefore the area of the ellipſe varies as $DC \times PF$.

Cor. 2. Hence $DC \times PF$* $= AC \times CB$, a conſtant quantity. * Fig. 12.

Cor. 3. Becauſe the △s SPY, HPZ, PFE are ſimilar we have,

$$SP : SY :: PE : PF$$
$$HP : HZ :: PE : PF$$
$$\therefore SP \times HP : SY \times HZ :: PE^2 : PF^2$$

but by pr. 6. $SY \times HZ = BC^2$, and by cor. 2. pr. 4. $PE^2 = AC^2$; therefore $SP \times HP = \frac{BC^2 \times AC^2}{PF^2} =$ (by the laſt cor.) DC^2.

PROP. XI.

To determine the diameter and chords of curvature paſſing through the center and focus of an ellipſe.

Let $PaVL$* be the circle of curvature to the point P, Qv an ordinate to the diameter PG, and QR parallel to PG, then by pr. 2. cor. 9. Pv $(QR) \times Gv :: Qv^2 :: PC^2 : CD^2$, $\therefore \frac{Qv^2}{QR} = \frac{CD^2 \times Gv}{PC^2} =$, * Fig. 20.

if Q moves to P, in the evaneſcent ſtate of the arc QP, $\frac{CD^2 \times 2PC}{PC^2} = \frac{2CD^2}{PC}$. But as Qv coincides in its evaneſcent ſtate with the arc QP, and conſequently with the circle of curvature, therefore by lem. 2. $\frac{Qv^2}{QR}$ ultimately becomes equal to the chord of curvature PV, $\therefore PV = \frac{2CD^2}{PC}$ = the chord of curvature paſſing through the center. Alſo if PFL be perpendicular to PR or DK, it muſt paſs through the center of the circle, conſequently by ſimilar △s PCF, PVL, $PF : PC :: PV \left(\frac{2CD^2}{PC}\right) : PL = \frac{2CD^2}{PF}$ = the diameter of curvature. Laſtly, if PS interſects the circle in a, and La be joined, by ſimilar triangles PFE, PaL, PE ($= AC$ by cor. 2. pr. 4.) $: PF :: PL \left(= \frac{2CD^2}{PF}\right) : Pa = \frac{2CD^2}{AC}$ = the chord of curvature paſſing through the focus.

PROP.

PROP. XII.

If a cone ABC* *be cut through its two ſides by a plane* PMNEZ, *the ſection thereof will be an ellipſe.*

* Fig. 22.

Biſect PE in K, and let a ſection LMI be drawn through K parallel to the baſe, and alſo another ſection $HMFG$ parallel to LMI, then will theſe ſections be circles, and by their property $HG \times GF = GN^2$, and $LK \times KI = KM^2$. Draw PQ, DE parallel to LI or HF, then by ſimilar triangles

$$PK : KL :: PG : GH$$
$$EK : KI :: EG : GF$$

$\therefore PK \times EK : KL \times KI :: PG \times GE : GH \times GF$, that is, $PK^2 : KM^2 :: PG \times GE : GN^2$, which is the property of the ellipſe by pr. 7.

Hence the ellipſe is called a conic ſection.

Of the HYPERBOLA.

* Fig. 23. **DEF. 1.** IF two right lines Sa, Hb*, interſecting each other in P, be made to revolve about H and S, ſo that $HP - SP$ may be a conſtant quantity, their common interſection P will deſcribe a curve called the *hyperbola*. And the hyperbola generated by the two lines Sp, Hp revolving about S and H, if $Sp - Hp = HP - SP$, is called an *oppoſite* hyperbola.

2. The points H, S, are called *foci*.

3. If A and M be the points where the curves interſect the line HS, the right line AM is called the *major axis*.

4. If AM be biſected in C (which is called the *center*) and BCN be drawn perpendicular to AM; then if from A

as a center with a radius *SC*, a circle be deſcribed cutting *BN* in *B* and *N*, *BN* is called the *axis minor*. And when *AM* = *BN*, the hyperbole is called *equilateral*.

5. If in like manner two hyberbolas *dBc*, *qNs*, be deſcribed about the two axes *BN*, *AM*, they are called *conjugate* hyperbolas.

6. A right line *LST* drawn perpendilar to *HS* and terminated by the curve, is called the *latus rectum*.

7. If *PCG* be a right line paſſing through the center, it is called a *diameter.*

8. If *QV* be drawn parallel to a tangent *P*, meeting *CP* produced in *V*, then *QV* is called an *ordinate*, and *GV*, *PV*, are called *abſciſſas*.

9. An *aſymptote* is a tangent to the curve at an infinite diſtance.

PROP. I.

The difference of the two lines HP, SP, *is equal to the major axis* AM, *or to* 2*AC*.

For conceiving the deſcribing points P, p, to come to A and M, and we have $HA - AS\ (= HP - PS = Sp - pH) = SM - MH$, take from the firſt and laſt of theſe the common part MA, and we have $HM - AS = SA - MH$, $\therefore 2MH = 2SA$, or $MH = SA$, and hence $HP - PS\ (= HA - AS = HA - HM) = MA$ or $2AC$.

Cor. 1. Hence $SC = CH$.

Cor. 2. Hence alſo by tranſpoſition $HP = 2AC + SP$.

PROP. II.

The rectangle under SA *and* SM *is equal to* BC^2.

For $BC^2 = BA^2 - CA^2 =$ (by conſtruction) $SC^2 - AC^2 = \overline{SC + AC} \times \overline{SC - AC} = \overline{SC + CM} \times SA = SM \times SA$.

PROP. III.

The latus rectum LT *is a third proportional to the axis major* AM *and the axis minor* BN.

By cor. 2. pr. 1. $HL = 2AC + LS$, hence $HS^2 + SL^2 (= HL^2) = 4AC^2 + 4AC \times LS + LS^2$, $\therefore HS^2 - 4AC^2 = 4AC \times LS$, but $HS = 2CS =$ (by conſtruction) $2AB$, $\therefore 4AB^2 - 4AC^2 = 4AC \times LS$, or $4BC^2 = 4AC \times LS$, $\therefore 2AC : 2BC :: 2BC : 2LS$, that is, $AM : BN :: BN : LT$.

PROP. IV.

Two right lines HQ*, SQ, *drawn from the foci to the curve make equal angles with the curve.*

* Fig. 24.

For draw SP indefinitely near to SQ and join HP, and draw Pa, Pb perpendicular to SQ and HQ, then will aQ, bQ be the decrements of SQ, HQ, which, as the difference of thoſe lines is conſtant, muſt be equal to each other, alſo the ∠s PbQ, PaQ are each right angles, and the ſide PQ is common to the two triangles PbQ, PaQ, $\therefore \angle HQP = \angle SQP$.

Cor. 1. Hence a tangent PV to the point P biſects the $\angle HPS$, for as the ∠s QSP, QHP are indefinitely ſmall, the ∠s SPV, HPV are equal to SQP, HQP

HQP reſpectively, which are equal by the propoſition.

* Fig. 25. Cor. 2. Hence if SP* be produced to meet a line CE parallel to the tangent at P in E, the part PE is equal to AC. For draw HI parallel to CE, meeting SP produced in I. Then becauſe PV is parallel to IH, the $\angle SPV = \angle PIH$, and $\angle VPH = \angle PHI$, but as $\angle SPV = \angle HPV$, $\therefore \angle PIH = \angle PHI$, and conſequently $PH = PI$. Alſo, as $SC = CH$ and CE is parallel to HI, $SE = EI$, hence $IE + EP = SE + EP$, that is, $IP = SP + 2PE$, $\therefore IP - PS = 2PE$, but $HP = IP$, $\therefore HP - SP = 2PE$; but by pr. 1. $HP - PS = 2AC$, $\therefore PE = AC$.

PROP. V.

* Fig. 26. *If two perpendiculars* HZ*, SY *be drawn to the tangent* PV, *they will meet it in the periphery of a circle whoſe diameter is* AM.

For produce SY to meet HP in a, then as the $\angle YPa = YPS$ by pr. 4. and $\angle SYP = aYP$, alſo PY common to the two Δs SPY, aPY, $\therefore SY = Ya$ and

$aP = SP$, hence Ha ($= HP - Pa$) $=$ $PH - PS =$ (by pr. 1.) AM. Now as $SC = CH$ and $SY = Ya$, CY is parallel to Ha, therefore $SH : SC :: Ha$ $(AM) : CY$, but $SC = \frac{1}{2}SH$, $\therefore CY =$ $\frac{1}{2}AM = AC$, and hence Y is in the periphery of a circle whoſe center is C and radius CA. For the ſame reaſon, if PS be produced to meet HZ produced in b, $CZ = \frac{1}{2}Sb =$ (as Hb is parallel to Sa) $\frac{1}{2}Ha = AC$ as before.

PROP. VI.

The rectangle under the perpendiculars SY, HZ, *is equal to* BC².

Let HZ interſect the circle in p and join Cp, then the $\angle pZY$ being a right angle, it muſt be in a ſemicircle, as Z and Y by the laſt pr. are in the circumference, and conſequently pCY muſt be a ſtraight line; hence in the triangles HCP, SCY, $HC = SC$, $Cp = CY$ and the $\angle HCp =$ $\angle SCY$, $\therefore Hp = SY$, and by the property of the circle $Hp \times HZ$ or $SY \times HZ =$ $HA \times HM =$ (by pr. 2.) BC^2.

Cor. 1. Since by pr. 4. $\angle SPY =$ $\angle HPZ$, the $\triangle$s SPY, HPZ are ſimilar,

$$\therefore SP : SY :: HP : HZ = \frac{SY \times HP}{SP},$$

hence $\frac{SY^2 \times HP}{SP} = HZ \times SY = BC^2$,

conſequently $SY^2 = BC^2 \times \frac{SP}{HP}$.

Cor. 2. Hence, as BC is conſtant, SY varies as $\sqrt{\frac{SP}{HP}}$, but $HP =$ (by cor. 2. pr. 1.) $2AC + SP$, $\therefore$ SY varies as $\sqrt{\frac{SP}{2AC + SP}}$, or $\frac{1}{SY}$ varies as $\sqrt{\frac{2AC + SP}{SP}}$.

Cor. 3. Since $4SY^2 = \frac{4BC^2 \times SP}{2AC + SP}$, and by pr. 3. $4BC^2 = 2AC \times L$, we have $4SY^2 = \frac{2AC \times L \times SP}{2AC + SP}$, hence $L \times SP : 4SY^2 :: L \times SP : \frac{2AC \times L \times SP}{2AC + SP}$ $:: 2AC + SP : 2AC$, conſequently, as $2AC + SP$ is greater than $2AC$, $L \times SP$ is greater than $4SY^2$.

PROP.

PROP. VII.

If PN* *be perpendicular to* MAN, *then* *Fig. 27. *will* $AN \times NM : NP^2 :: AC^2 : BC^2$.

Take $AD = PS$, and then $SP = CD - CA$, and by pr. 1. $HP = CD + CA$, alſo $NH = NC + CH = NC + CS$, and $SN = NC - CS$; hence by Eucl. 47. 1. $\overline{CD + CA}^2 = PN^2 + \overline{NC + CS}^2$, and $\overline{CD - CA}^2 = PN^2 + \overline{NC - CS}^2$, ſubtract this latter equation from the former, and we have $CD \times CA = CN \times CS$, $\therefore CD = \dfrac{CN \times CS}{AC}$; hence $PS = \dfrac{CN \times CS}{CA} - CA = \dfrac{CN \times CS - CA^2}{CA}$, $\therefore$ by Eucl. 47. 1. (as $NS = NC - CS$)

$$\frac{CN^2 \times CS^2 - 2CN \times CS \times CA^2 + CA^4}{CA^2}$$

$= PN^2 + NC^2 - 2NC \times CS + CS^2$, which is the ſame equation as that for the ellipſe in pr. 7. and conſequently there reſults from it the ſame concluſion, that is, $AN \times NM : PN^2 :: AC^2 : BC^2$.

Cor. 1. Hence $PN = \frac{BC}{AC} \times \sqrt{AN \times MN}$; conſequently, if upon the ſame axis major, to any other axis minor bg, an hyperbola vAw be drawn, and p be the point where it interſects the ordinate NP, then $Np = \frac{bC}{AC} \times \sqrt{AN \times MN}$, and hence $NP : Np$ as $BC : bC$, that is, in a given ratio, and therefore, for the ſame reaſon as in the parabola, the area APS : area ApS in the ſame given ratio of $BC : bC$.

Cor. 2. If $L =$ the latus rectum, and GB be the axis minor, then by pr. 3. $AM : BG :: BG : L$, hence $AM : L :: AM^2 : BG^2 :: AC^2 : BC^2 :: AN \times NM : PN^2$, $\therefore L = \frac{AM \times PN^2}{AN \times NM} = \frac{2AC \times PN^2}{AN \times NM}$.

Cor. 3. If $AC = a$, $BC = b$, $CN = x$, $NP = y$, then we have $\overline{x - a} \times \overline{x + a}$ $(x^2 - a^2) : y^2 :: a^2 : b^2$, $\therefore y^2 = \frac{a^2}{b^2} \times \overline{x^2 - a^2}$.

Cor. 4. Becauſe $SP = CD - CA$, and

$HP = CD + CA$, therefore $\frac{SP + HP}{2} = CD$, hence $CN \times CS$ ($= CA \times CD = CA \times \frac{SP + HP}{2}$.

Cor. 5. As AC and BC are conſtant in the ſame hyperbola, $AN \times NM$ varies as PN^2.

PROP. VIII.

If PT *be a tangent at* P, *then will* CN : CA :: CA : CT.

For as by pr. 4. PT biſects $\angle SPH$, $HP : SP \mathrel{::} HT : TS$, and conſequently $HP + PS : HP - PS \mathrel{::} HT + TS : HT - TS$, that is, $HP + PS : 2AC \mathrel{::} HS : 2TC$, or $AC : TC \mathrel{::} \frac{HP + PS}{2} : \frac{HS}{2} (CS) \mathrel{::} AC \times \frac{HP + PS}{2} : AC \times CS \mathrel{::} CN \times CS$ (by cor. 4. p. 7.) $: AC \times CS \mathrel{::} CN : AC$, hence $CN : CA \mathrel{::} CA : CT$.

Cor. Hence if $AC = a$, $BC = b$, CN

$= x$, $NP = y$, then $x : a :: a : CT = \frac{a^2}{x}$, hence $NT = x - \frac{a^2}{x} = \frac{x^2 - a^2}{x}$, conſequently (by cor. 3. pr. 7.) $PN^2 : TN^2 :: \frac{b^2}{a^2} \times \overline{x^2 - a^2} : \frac{\overline{x^2 - a^2}|^2}{x^2} :: b^2 x^2 : a^2 x^2 - a^4$.

PROP. IX.

To draw an aſymptote to a hyperbola.

Fig. 29. By cor. to pr. 8. $CT^* = \frac{a^2}{x}$; make x infinite and $CT = 0$, conſequently a tangent to the curve at an infinite diſtance paſſes through the center. Alſo by the ſame cor. $PN^2 : NT^2 :: b^2 x^2 : a^2 x^2 - a^4$ :: (when x becomes infinite) $b^2 x^2 : a^2 x^2 :: b^2 : a^2$, hence $PN : TN :: b : a$, or as T now coincides with C, $PN : CN :: b : a$; but as P is now a point in the tangent at an infinite diſtance, we have by ſimilar △s, $PN : CN :: Aa : AC\ (a)$, therefore $Aa = b$; hence if Aa be erected perpendicular to AC and equal to b, and CaZ be drawn, it ſhall be the aſymptote.

Hence, if on the other ſide of *AC*, a perpendicular *Ab* be erected equal to *AC*, and *CbL* be drawn, it ſhall be an aſymptote to the other part; and conſequently if theſe aſymptotes be produced on the other ſide of the center, they ſhall be aſymptotes to the oppoſite hyperbola.

Cor. 1. Hence alſo theſe aſymptotes will be aſymptotes to the conjugate hyperbolas; for perpendiculars to *BG*, from *B* and *G*, equal to *CA*, will terminate at *a* and *b*, and conſequently by the pr. *CaZ*, *CbL* will be aſymptotes to theſe hyperbolas.

Cor. 2. The aſymptotes make equal ∠s with the axis major.

PROP. X.

If PN* *be produced to meet the aſymptotes in* Q *and* q, *then* $PQ \times Pq = Aa^2$.

* Fig. 30.

For by ſimilar △s CAa, CNQ,

$$CN^2 : CA^2 :: NQ^2 : Aa^2$$

but $CA^2 : CN^2 - CA^2 :: Aa^2 : PN^2$ by c. 3. pr. 7.
$\therefore CN^2 : CN^2 - CA^2 :: NQ^2 : PN^2$; and dividendo, $CN^2 : CA^2 :: NQ^2 : NQ^2 - PN^2$ $(\overline{NQ + PN} \times \overline{NQ - PN}) :: NQ^2 : Pq \times PQ$,

PQ, from which, and the firſt proportion, we have $Pq \times PQ = Aa^2$.

Cor. 1. For the ſame reaſon $pQ \times pq = Ab^2$, or Aa^2, and therefore $PQ \times Pq = pq \times pQ$.

Cor. 2. Hence if $WVvw$ be drawn parallel to Qq, $VW \times Vw = Aa^2$, therefore $VW \times Vw = PQ \times Pq$.

PROP. XI.

* Fig. 31. *If a line* XVpx* *be drawn in any poſition, cutting the two aſymptotes in* X, x, *and the curve in* V, p, *then will* VX × Vx = pX × px.

For through V and P draw two lines $WVvw$, QPq, perpendicular to the axis, and through P draw RPr parallel to Xx; then on account of the parallel lines Ww, Qq and Xx, Rr, the △s QPR, Pqr, XVW, Vxw are ſimilar,

$\therefore\ VX : VW :: PR : PQ$

and $Vx : Vw :: Pr : Pq$, hence $VX \times Vx : VW \times Vw :: PR \times Pr : PQ \times Pq$, but by cor. 2. pr. 10. $VW \times Vw = PQ \times Pq$, $\therefore VX \times Vx = PR \times Pr$. Conceive now the line RPr to move parallel to itſelf, until it becomes a tangent LEM at the point E, then PR becomes EL and Pr becomes EM; hence $VX \times Vx = EL \times EM$. For the ſame reaſon $pX \times px = EM \times EL$, $\therefore VX \times Vx = pX \times px$.

Cor. 1. Hence $VX = px$. For $VX \times Vx = pX \times px$, that is, $VX \times \overline{Vp + px} = px \times \overline{pV + VX}$, take therefore from both ſides $VX \times px$, and we have $VX \times Vp = px \times pV$, that is, $VX = px$.

Cor. 2. Hence if we ſuppoſe Xx to move parallel to itſelf and coincide with LM, we ſhall have $LE = EM$.

Cor. 3. Let Xx cut CE produced in O, then becauſe of the parallel lines LM, Xx, as $LE = EM$, we have $XO = xO$, but by cor. 1. $VX = px$, $\therefore VO = pO$, conſequently the diameter CEZ biſects all its ordinates.

Cor. 4. Hence $VX \times Vx = EL^2$.

Cor. 5. As $VX = px$, $\therefore Xp = Vx$, conſequently $XV \times Xp = EL^2 = EL \times EM$.

PROP.

PROP. XII.

If BCb *be drawn parallel to* LM, *and from* E, EB, Eb *be drawn parallel to* CM, CL *reſpectively, then will* $BC = bC = LE$ *or* ME.

For as CM is parallel BE, and CB to EM, $CBEM$ is a ▭, $\therefore CB = EM$; for the ſame reaſon $Cb = EL$, but $LE = EM$, $\therefore CB = Cb$.

Cor. 1. As by cor. 4. pr. 11. $VX \times Vx = EL^2$, $\therefore VX \times Vx = BC^2$.

Cor. 2. As Ca is parallel to BE and $BC = Cb$, $\therefore Ea = ab$.

Def. 10. Bb is called a *conjugate* diameter to the diameter CEZ.

PROP. XIII.

If CE *be produced to meet the oppoſite hyperbola in* D, *then will* $DO \times EO : OV^2 :: CE^2 : CB^2$.

For by ſimilar △s CEL, COX, CE : EL

$EL :: CO : OX = \frac{EL \times CO}{CE}$, which is $=$ Ox, as $OX = Ox$; hence $VX = OX - OV$ $= \frac{EL \times CO}{CE} - OV = \frac{EL \times CO - CE \times OV}{CE}$, and $Vx = Ox + VO = \frac{EL \times CO}{CE} + VO$ $= \frac{EL \times CO + CE \times OV}{CE}$, ∴ by cor. 1. pr. 12. $\frac{EL^2 \times CO^2 - CE^2 \times OV^2}{CE^2} = CB^2$, $\therefore CE^2 \times CB^2 =$ (as $EL = CB$) $CB^2 \times CO^2$ $- CE^2 \times OV^2$, conſequently $CE^2 \times OV^2 =$ $CB^2 \times \overline{CO^2 - CE^2} = CB^2 \times \overline{CO + CE} \times$ $\overline{CO - CE} = CB^2 \times \overline{CO + CD}\ (DO) \times$ EO; hence $DO \times EO : OV^2 :: CE^2 : CB^2$.

PROP. XIV.

If DV, SE*, *be drawn parallel to the aſymptote* LC, *and* ET *parallel to* CZ, *then* DV × DC = SE × TE.

* Fig. 32.

For draw VR parallel to ZC, then the △s XVD, LES, VxR, EMT, being ſimilar, we have

$DV : SE :: XV : LE$
$VR : ET :: Vx : EM$
$\therefore DV \times VR : SE \times ET :: XV \times Vx : LE \times EM$, but by cor. 5. pr. 11. $XV \times Vx = LE \times EM$, $\therefore DV \times VR = SE \times ET$, that is, as $VR = DC$, $DV \times DC = SE \times ET$.

Cor. 1. Becauſe $LE = EM$ by cor. 2. pr. 11. and SE is parallel to CL, therefore $LS = SC$, or $SC = \frac{1}{2}CL$.

Cor. 2. Hence $DV \times DC$ is a conſtant quantity, and therefore DV is inverſely as DC.

Cor. 3. If the point E coincides with A*, then the $\angle ACT = \angle ACS$ by cor. 2. pr. 9. but as $ATCS$ is ▭, the $\angle ACT = \angle CAS$, $\therefore \angle CAS = \angle ACS$, conſequently $AS = SC = AT$.

* Fig. 33.

Cor. 4. Hence $CD \times DV = SA^2$.

Cor. 5. As $\triangle$s BCS, CGT are ſimilar, and $BC = CG$, $\therefore GT = CS = TA$. And for the ſame reaſon $BS = SA$.

PROP. XV.

If LMN *be drawn parallel to* CD, *then will* LM = MN.

For by cor. 4. pr. 14. $LM \times MC = TA^2$, and $NM \times MC = TG^2$, but $TA = TG$ by cor. 5. pr. 12. $\therefore LM = MN$.

Cor. Hence by pr. 12. cor. 2. *NCa* is a conjugate diameter to *LCb*, and conſequently parallel to a tangent at *L*; and for the ſame reaſon *LCb* is a conjugate diameter to *NCa*, and therefore parallel to a tangent at *N*.

PROP. XVI.

All the parallelograms circumſcribing any two diameters of an hyperbola, are equal.

Let *abcd** be a ▭ circumſcribing the two diameters *DK*, *PG*, join *PD*, and draw *Dn* perpendicular to the aſymptote *CZ*. Now by pr. 15. and its cor. the aſymptote *CZ* biſects *DP*, which is the diagonal of the ▭ *CDaP*, conſequently the angle *a* muſt be ſituated in that

* Fig. 34.

aſymptote. Alſo by pr. 15. DmP (m being the point where DP interſects CZ) is parallel to the aſymptote CL and conſequently given in poſition, or the $\angle Dmn$ is conſtant, and therefore Dm is to Dn in a given ratio; but by pr. 12. cor. 2. Dm is inverſely as Cm, therefore Dn is inverſely as Cm, and by cor. 1. pr. 14. $Cm = \frac{1}{2}Ca$, therefore Dn is inverſely as Ca, and conſequently $Dn \times Ca$ is conſtant; but $Dn \times Ca$ is equal to the area of the $\square\, DaPC$, therefore the $\square\, DaPC$ is a conſtant quantity, but $\square\, DaPC$ is a fourth part of the $\square\, abcd$, therefore the $\square\, abcd$ is a conſtant quantity.

Cor. 1. Hence if PF be drawn perpendicular to CD, then $CD \times PF$ ($= \square\, CPaD = \square\, CAvB$) $= AC \times BC$, a conſtant quantity.

* Fig. 25. Cor. 2. Becauſe the $\triangle$s SPY*, HPZ, FPE are ſimilar, we have

$$SP : SY :: PE : PF$$

$$HP : HZ :: PE : PF$$

$$\therefore SP \times HP : SY \times HZ :: PE^2 : PF^2,$$

but by pr. 6. $SY \times HZ = BC^2$, and by cor. 2. pr. 4. $PE^2 = AC^2$, therefore SP

$\times HP = \frac{BC^2 \times AC^2}{PF^2} =$ (by the laſt cor.) CD^2.

PROP. XVII.

To determine the diameter and chords of curvature paſſing through the focus and center.

Let $PaLV$* be a circle of curvature to the point P, Qv an ordinate to the diameter PG, and QR parallel to Pv, then by pr. 13. $Pv\,(QR) \times vG : Qv^2 :: PC^2 : CD^2$, $\therefore \frac{Qv^2}{QR} = \frac{vG \times DC^2}{CP^2} =$, if Q moves up to P, in the evaneſcent ſtate of the arc QP, $\frac{CD^2 \times 2PC}{CP^2} = \frac{2CD^2}{PC}$. But as Qv coincides in its evaneſcent ſtate with the arc QP, and conſequently with the circle of curvature, therefore by lem. 2. $\frac{Qv^2}{QR}$ ultimately becomes equal to the chord of curvature PV, $\therefore PV = \frac{2CD^2}{PC} =$ the chord of curvature paſſing

* Fig. 35

through

through the center. Alſo if LPF be perpendicular to PR or CD, it muſt paſs through the center of the circle, and hence by ſimilar $\triangle$s PVL, PCF, $PF : PC :: PV\left(\frac{2CD^2}{PC}\right) : PL = \frac{2CD^2}{PF} =$ the diameter of curvature. Laſtly, if PS (produced if neceſſary) interſects the circle in a, and La be joined, by ſimilar $\triangle$s PEF, PLa, PE ($= AC$ by cor. 2. pr. 4.) $: PF :: PL\left(\frac{2CD^2}{PF}\right) : Pa = \frac{2CD^2}{AC}$ = the chord of curvature paſſing through the focus.

PROP. XVIII.

* Fig. 36. *If a cone* RLW* *be cut through the ſide* RL *by a plane* AFG, *which being produced will meet the other ſide of the cone* MR *produced in any point* M, *the ſection will be an hyperbola.*

Let $XPYV$ be a ſection of the cone parallel to the baſe LW, which being a circle $XN \times NY = NP^2$; draw RQ parallel to the diameter YX, then will the $\triangle$s RMQ, NMY, and AQR, AXN be ſimilar, hence

$$AQ : QR :: AN : NX$$
$$MQ : QR :: MN : NY, \quad \therefore$$
$$AQ \times MQ : QR^2 :: AN \times MN : NX \times NY \text{ or } NP^2,$$

but the two firſt terms are conſtant quantities, therefore $AN \times NM$ is to PN^2 in a conſtant ratio, and conſequently by cor. 5. pr. 7. the ſection is an hyperbola, whoſe major axis is AM.

Hence the hyperbola is called a conic ſection.

PROBLEM.

Given the diſtance of any point of a conic ſection from the focus, the latus rectum, and the poſition of the tangent, to determine the conic ſection.

The diſtance SP* and the poſition of the tangent being given, the perpendicular SY will be given; now by cor. 1. pr. 6. of the ellipſe and hyperbola, $\frac{SY^2 \times HP}{SP} = BC^2$, alſo $2AC = HP \pm SP$, according as the curve is an ellipſe or hyperbola; therefore the latus rectum L (=

* Fig^s. 12. 26.

$L\left(=\frac{4BC^2}{2AC}\right)=\frac{4SY^2\times HP}{SP\times\overline{HP\pm SP}}$, conſequently $HP=\frac{\pm L\times SP^2}{4SY^2-L\times SP}$; now in the ellipſe by cor. 3. pr. 6. $L\times SP$ is leſs than $4SY^2$; when therefore $L\times SP$ is leſs than $4SY^2$, draw PH* on the ſame ſide of the tangent with SP, making an angle $HPZ=\angle SPY$, and take $HP=\frac{L\times SP^2}{4SY^2-L\times SP}$, and H will the other focus; alſo $SP+PH=$ the major axis; and as HP is known, HZ will be given, conſequently $SY\times HZ$ or BC^2 will be known, and therefore the conic ſection is determined. In the hyperbola by cor. 3. pr. 6. $L\times SP$ is greater than $4SY^2$; when therefore $L\times SP$ is greater than $4SY^2$, draw PH* on the other ſide of the tangent in reſpect to SP, making an angle $HPV=\angle SPV$, and take $HP=\frac{-L\times SP^2}{4SY^2-L\times SP}$ or $\frac{L\times SP^2}{L\times SP-4SY^2}$, and H will be the other focus; alſo $HP-SP=$ the major axis, and as HP is known, HZ will

*Fig. 12.

*Fig. 26.

will be given, conſequently $SY \times HZ$ or BC^2 will be known, and therefore the conic ſection is determined. In the parabola by cor. 3. pr. 7. $L \times SP = 4SY^2$, and hence the following conſtruction. Make the angle $TPM^* = TPS$, and take $PM = PS$, draw DME perpendicular to PM, through S draw ZSL parallel to PM, and biſect SL in A, which will be the vertex of the parabola, from which the parabola itſelf is determined.

* Fig. 1.

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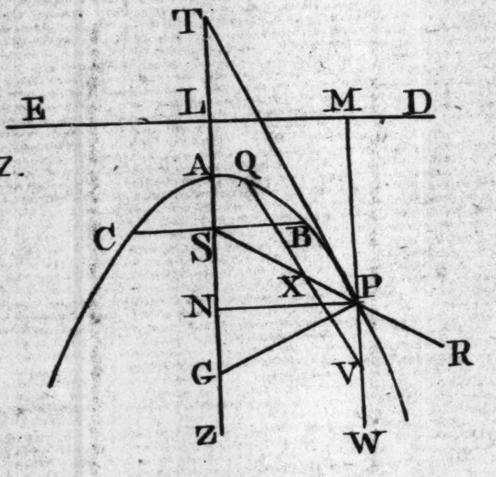
ERRATA.

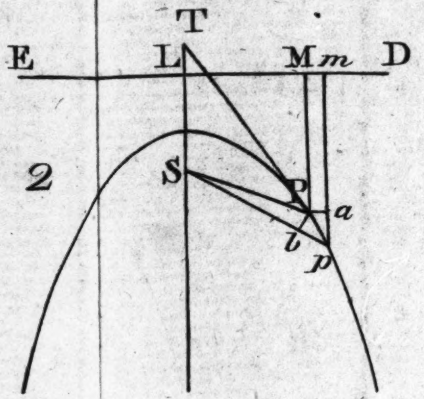
Page 7. line 8. *for* let on, *read* drawn to.
17. l. 18. *for* the, *read* any.
20. l. 17. *after* equation, *read* from the.
23. l. 8. *for* of the, *read* to the.
24. l. *penult*. *for* Δ, *read* Δs.
26. l. 17. *for* be, *read* to be.
31. l. 7. *for* HMFG, *read* HNFG.
40. l. *penult*. *for* $\frac{a^2}{b^2}$ *read* $\frac{b^2}{a^2}$.
41. l. 2. *after* CA × CD, *put*).
42. l. 6. *for* a, *read* an.
46. l. 6. *for* parallel, *read* parallel to.
48. l. 19. *for* As, *read* As the.

PL, 1

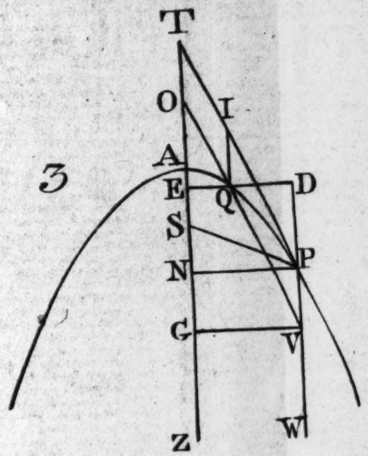
Fig. 1.



2



3



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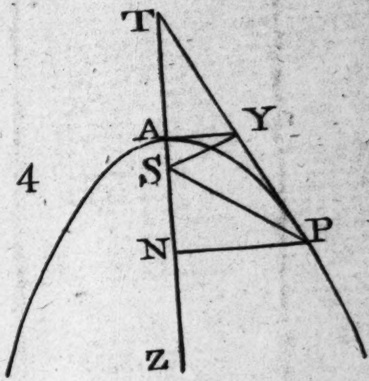
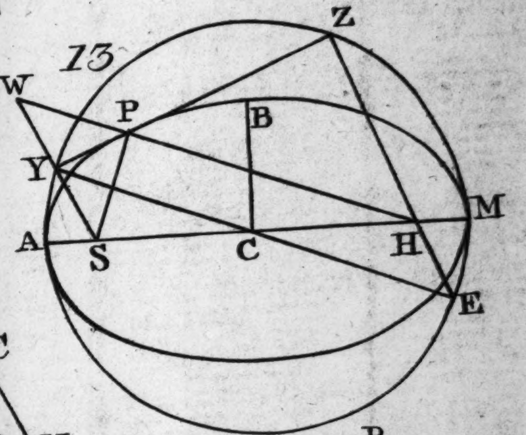


13



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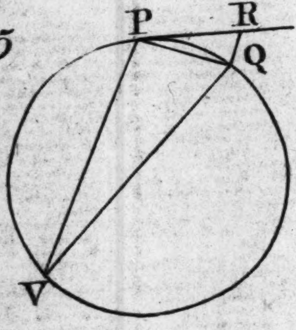

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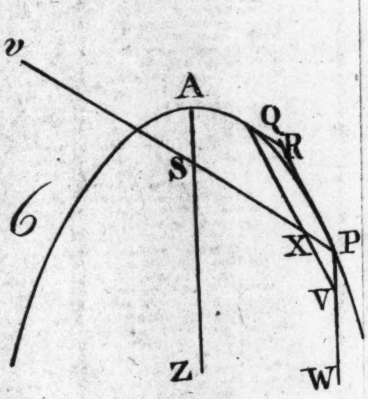

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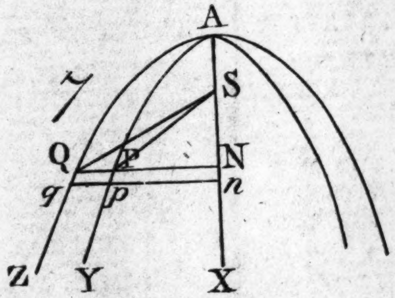

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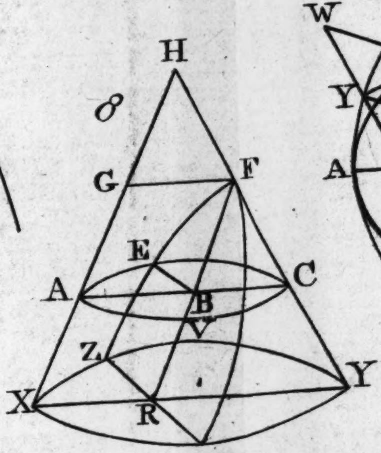

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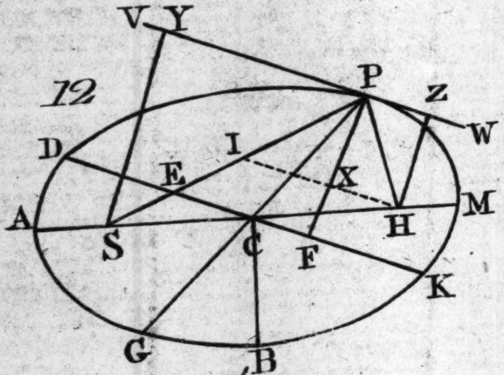

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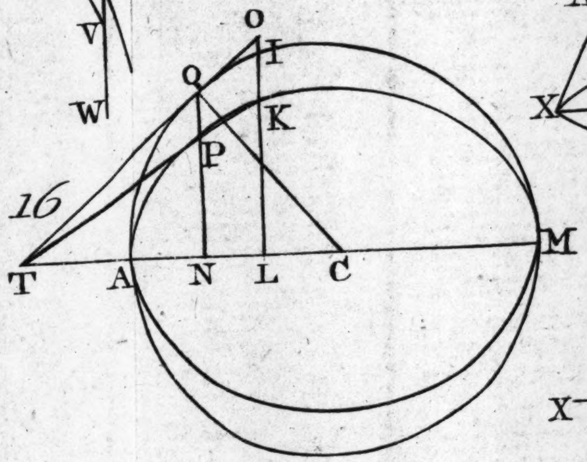

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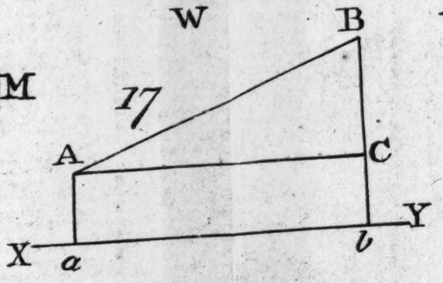

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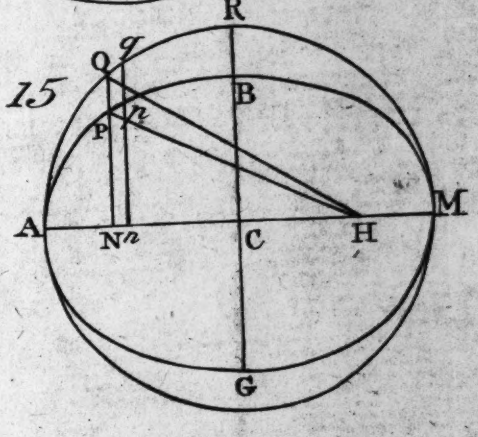

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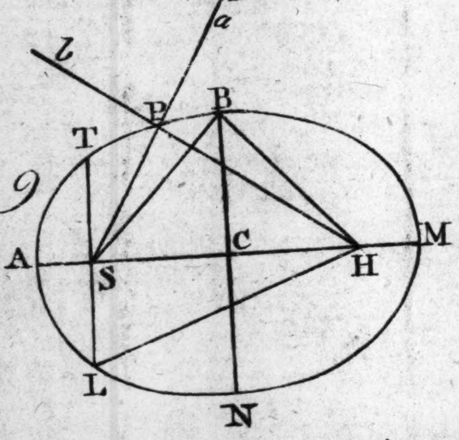

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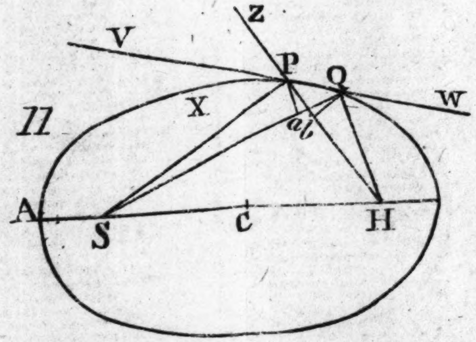

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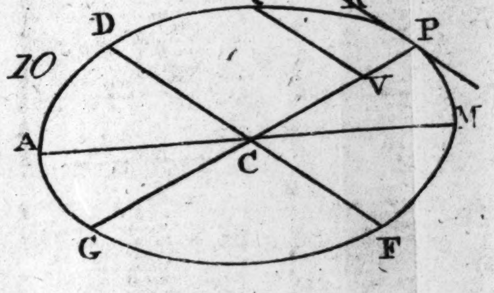

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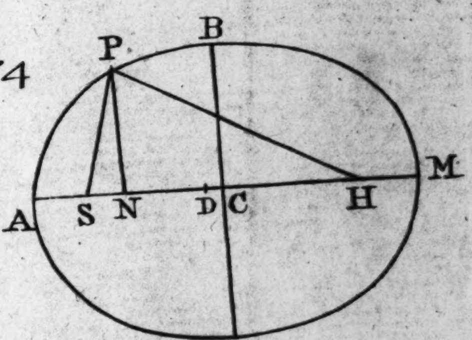

Pl, II

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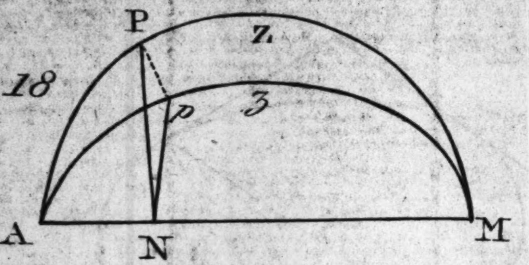

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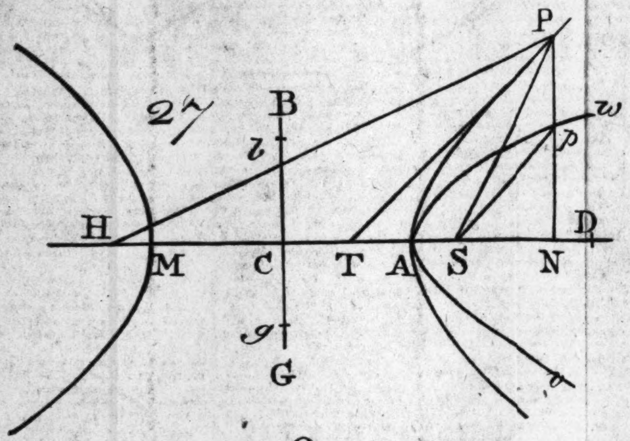

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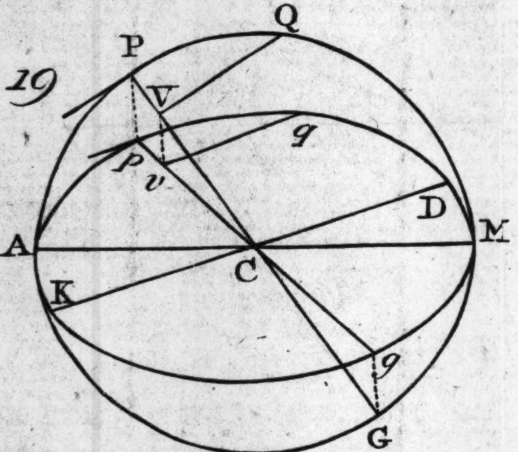

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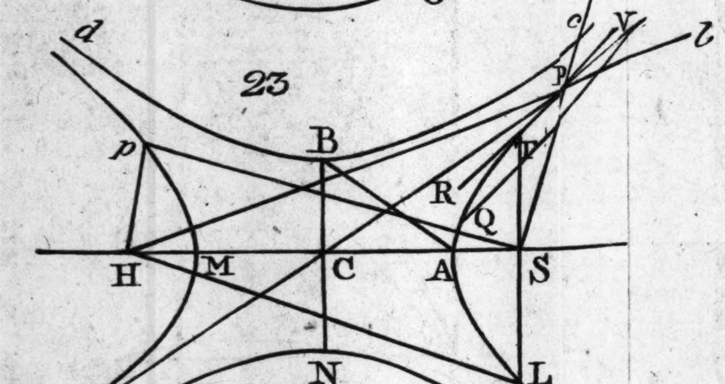

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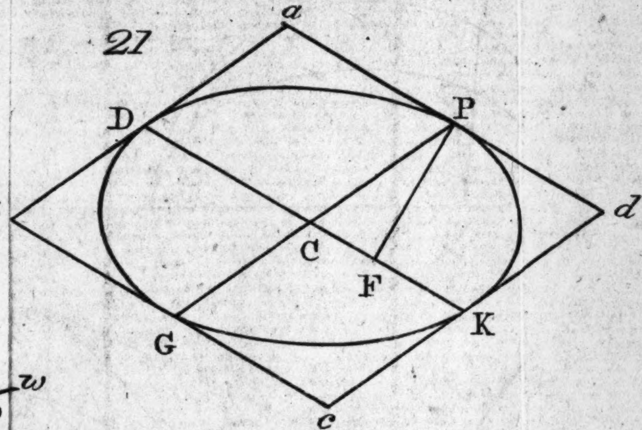

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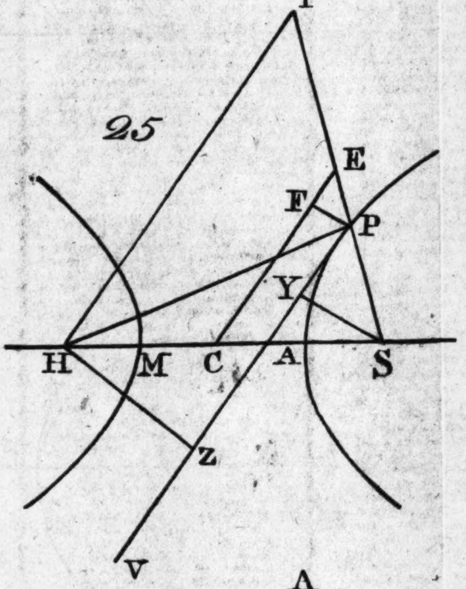

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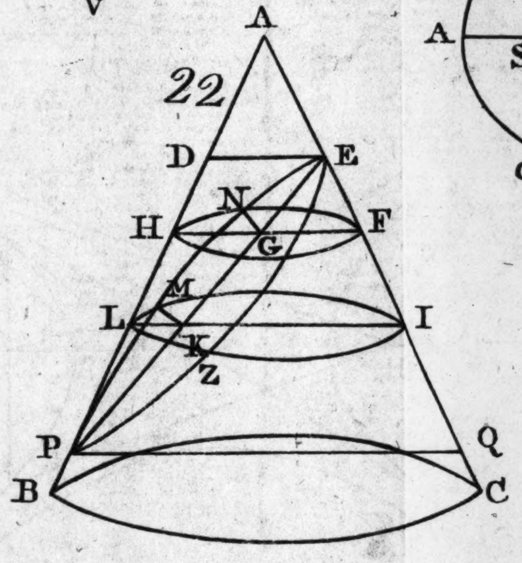

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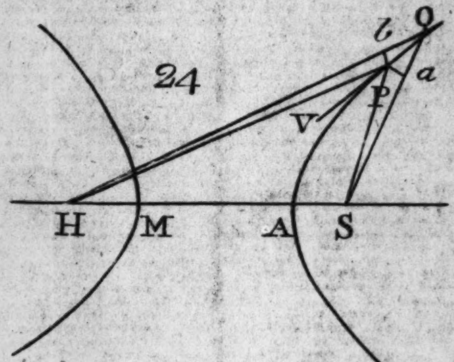

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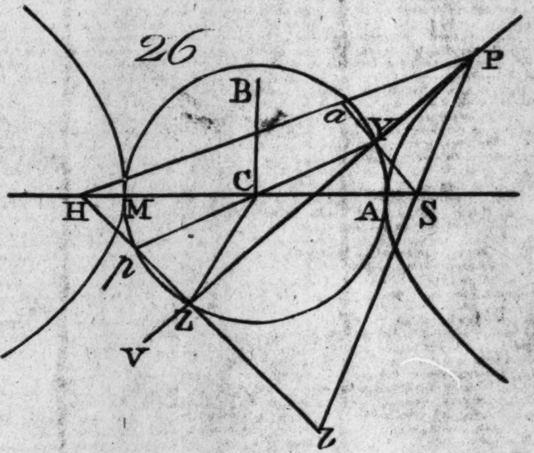

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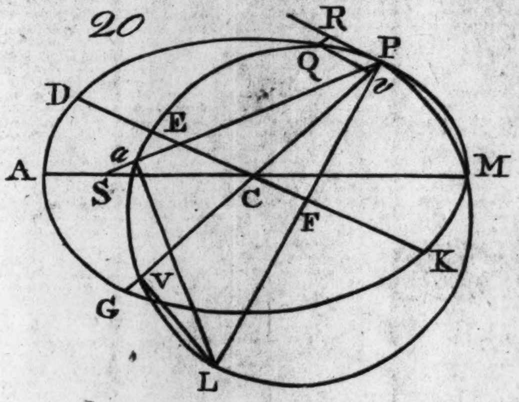

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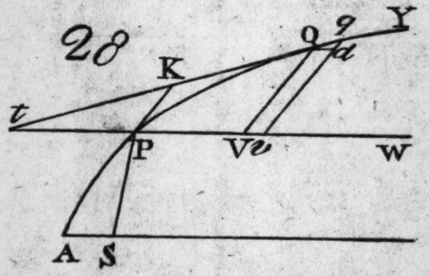

PL, III

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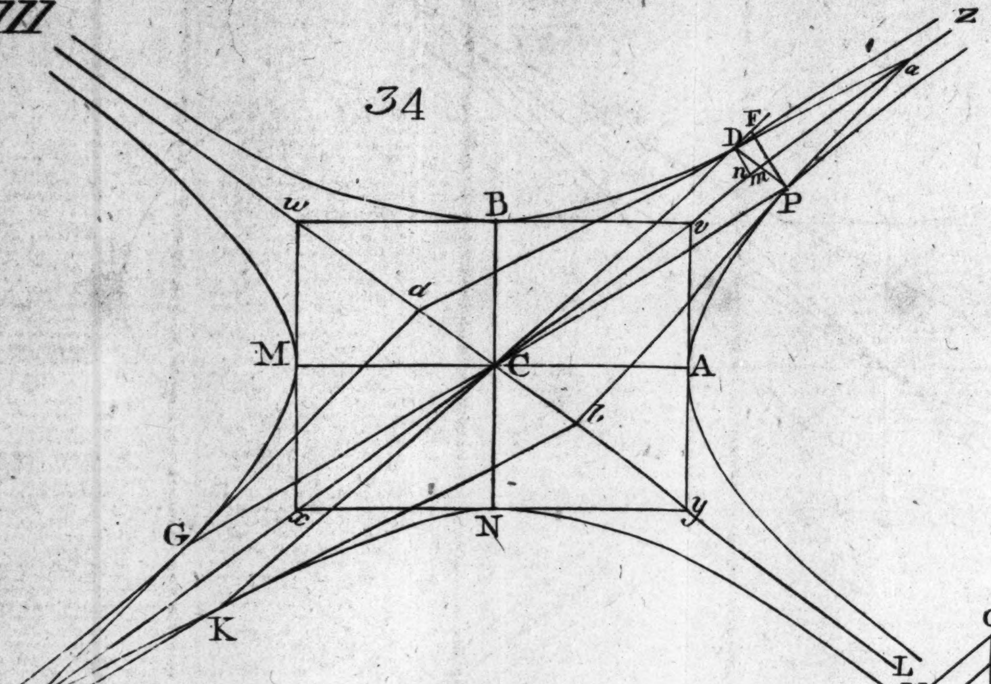

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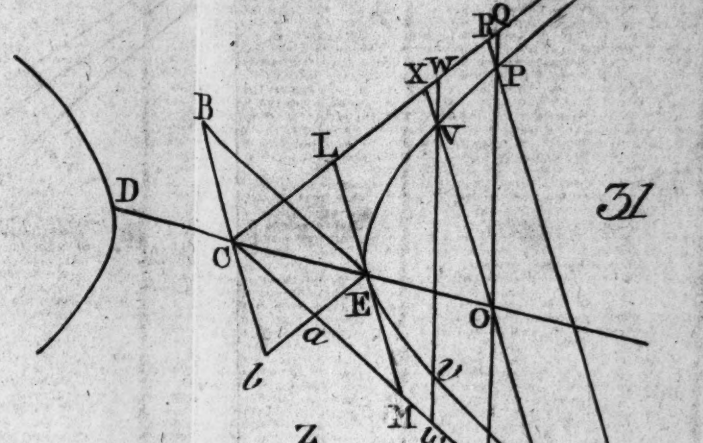

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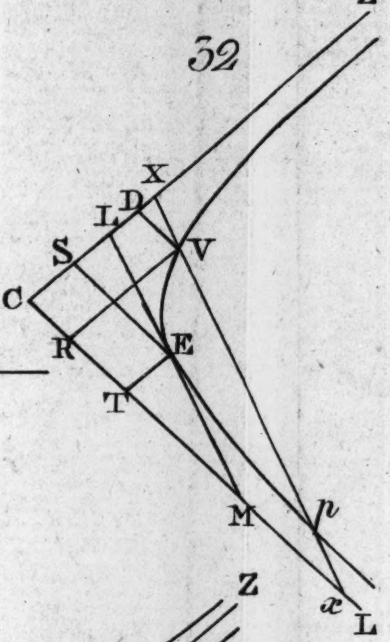

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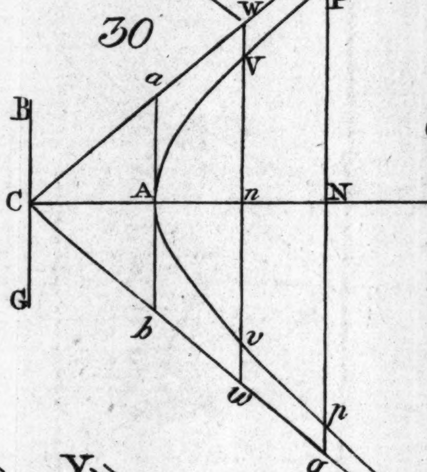

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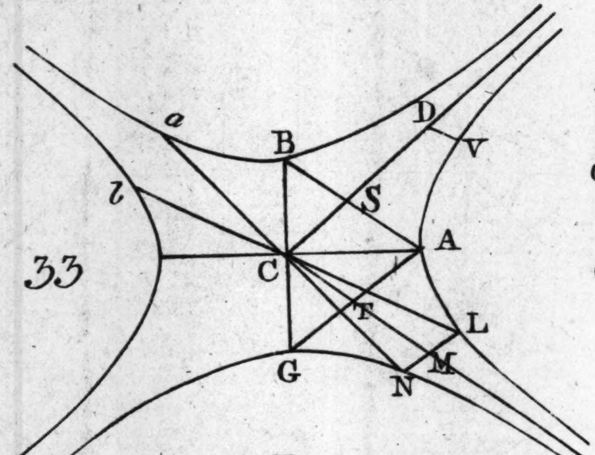

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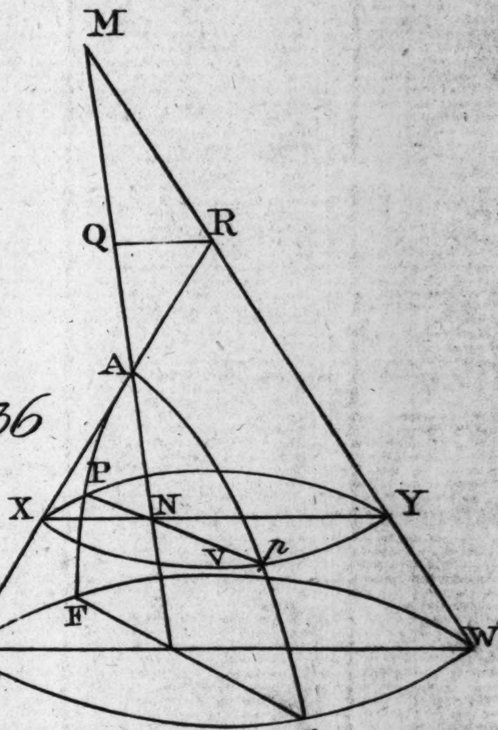

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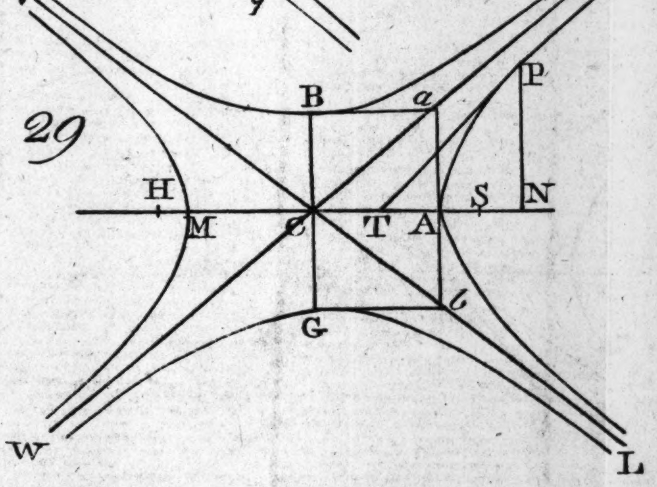

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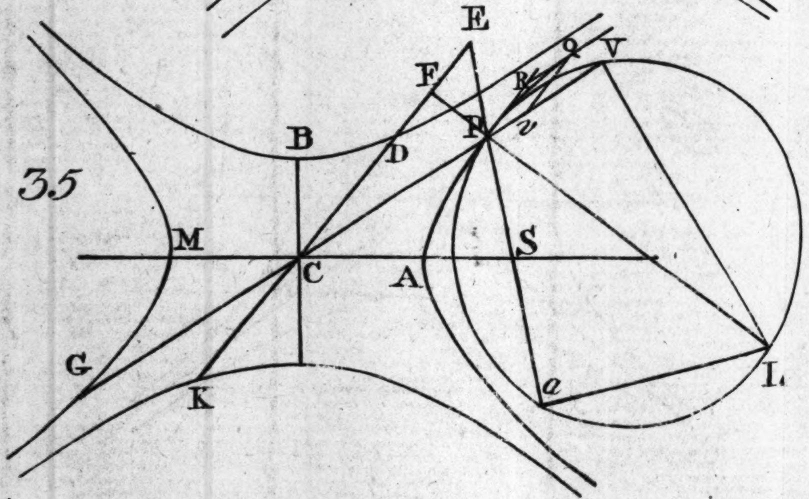
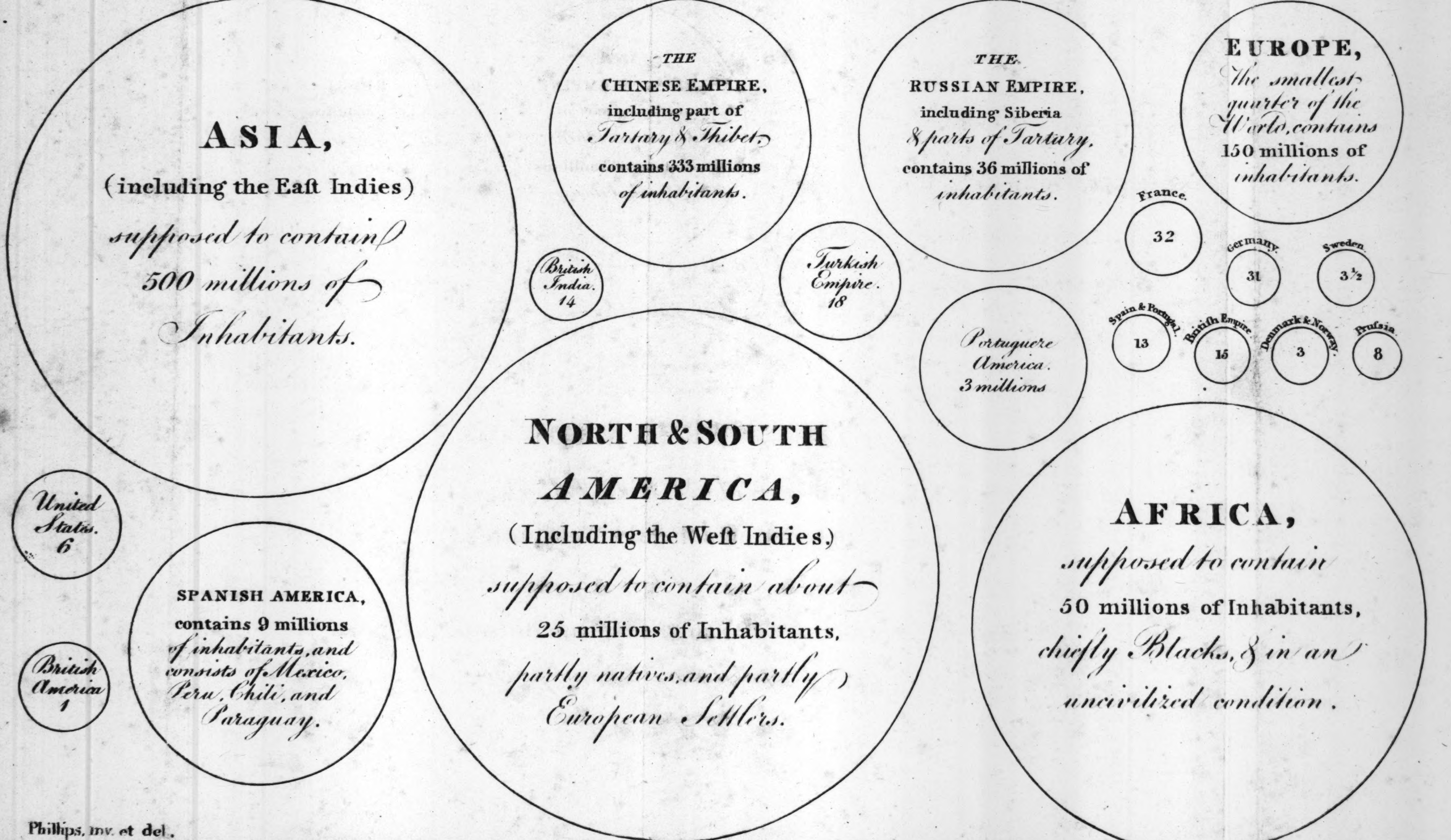


LINEAR GEOGRAPHY.

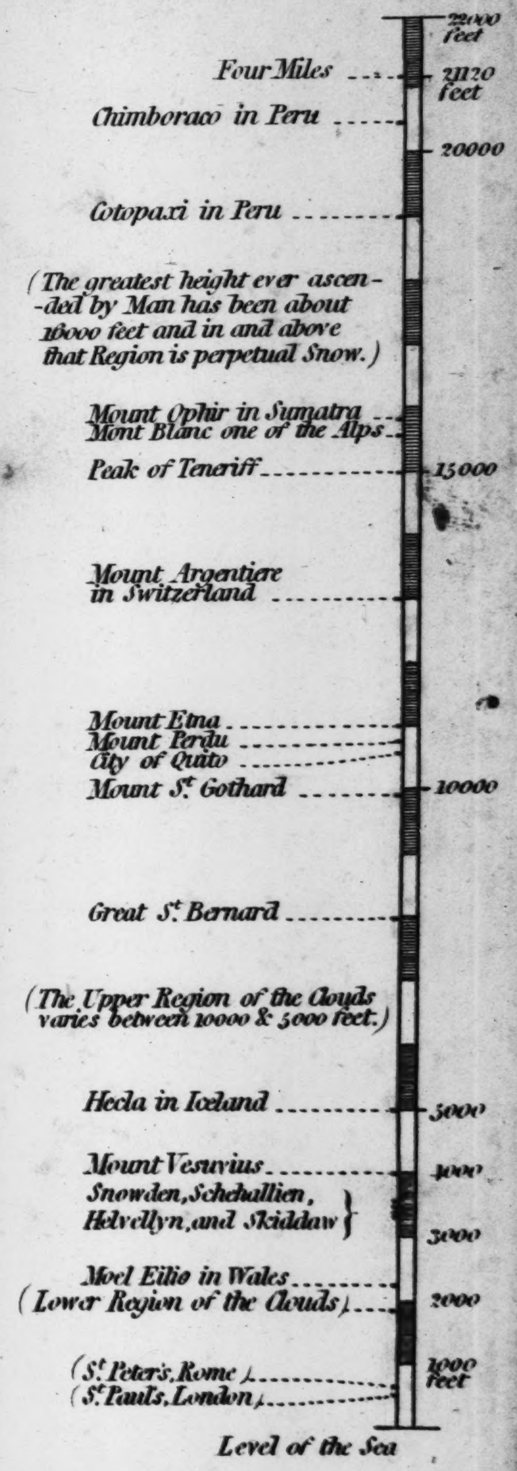
Supposing the surface of the whole Globe to be represented by a circle the diameter of which is One foot, then the diameter of a circle corresponding with the surface of the sea will be $10\frac{1}{3}$ inches; and the diameter of a circle corresponding with the surface of the land will be six inches.

In proportion to these circles, which, for the sake of perspicuity, should be drawn by the pupil, the circles represented beneath will nearly correspond with the sizes of the countries expreſsed.

N.B. The figures annexed to each denote the population in millions.



SCALE
of the heights of the Principal Mountains in THE WORLD.



Phillips, inv. et del.

Hemſley ſcripsit.

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